The theoretical analysis of the creation of forces on a thin foil at all angles of attack and velocities.

1. Introduction.

I am in the process of developing a sailing boat that works in a novel manner and has various benefits over current boats and is designed to at great speed. I have working prototypes etc and this year I was hoping to produce a final prototype that would demostrate my latest thinking and all the concepts together. However, at the time of writing, I am locked down in a country the otherside of the globe and can't return because British Airways have cancelled all my flights. However, that has given me time to hone the theoretical side of my invention.

The boat achieves power via an aerofoil. It has several novel design features, one of which is an abiltiy to vary its angle of attack from zero to ninety. Now, to find the force that a certain sized aerofoil would create at any angle of attack I thought would be a lookup-able figure subject to years of research and usage. I found however, that no such a thing exists! There are plenty of formulae available for wings; for a start the formula $L = (1/2)\ d\ V^2\ A\ CL$ [2], plus all the background maths for a lifetime of study. But this only works for small angles of attack and change the angle of attack and the whole thing must be reworked with a new lift coefficient, not at all useful for my purposes. Secondly, there is a very similar wind resistance or drag formula [3] which is very useful of course, for working out wind forces on advertising hoardings and the like, but that *only* applies to wind at 90 degrees and not very helpful for my needs. So, as I said, with time on my hands I thought I would try to go back to basic principles and see if I could come up with a new approach that solves my problems. And maybe some of other people's problems too, as ordinary sails are aerofoils that operate with differnet angle of attack, up wind and down wind.

Various other authors [1] [2] have produced experimental results and these will be used to check the veracity of this theory.

2. The forces on the lower surface of a theorectical thin, flat plate foil.

I have developed some mathematics to predict the behaviour and forces created by a planing surface as distinct from a wing or sail or other aerofoil (that may be published too). The workings of that are not totally relevant to a foil but for one thing, and that is the force created is related to the angle of attack. Referring to diagram (1) in which the sail or aerofoil or hydrofoil is replaced for simplicity's sake by, perfectly thin, flat plate AB subject to a fuid moving DC. As mentioned above, the force hitting AB is related to the angle of attack (AoA or ø); in fact sin(ø), the bigger the angle the bigger the mass flow and force along CE. And this is where my approach needs to differ from conventional wing theory which breaks this force down to two vectors *Lift* and *Drag*, up and back, at 90° to each other. I am only interested in total force in the perpendicular direction of the foil, although the two vectors do bear a relationship. So the lower surface deflects the air downwards according to the sine of the AoA and in accordance to Newton's third law the foil receives a reaction or upward force and this will be calculated along CE.

3. The upper surface force on a foil.

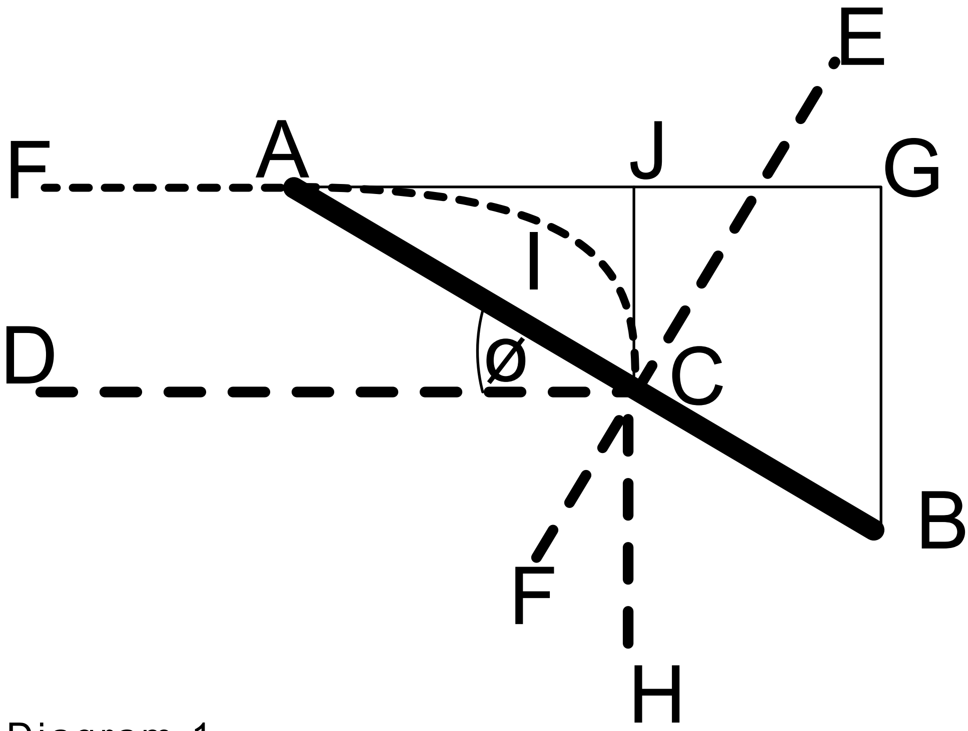


Diagram 1.

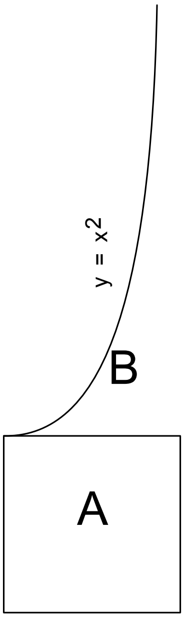


Diagram 2.

The upper surface is more complicated. So how does the upper surface of a flat plate create force, and unintuitively, more than the lower side?

The fluid moves past the foil along DC and area of fluid above AGB is lost and this is replaced from above, this fall of the air again supplies force to win according to Newton's third law. Well t *should* be the same as the lower surface effect but it is more as all the fluid above the wing falls down as well. Unfortunately, the lift is not as great as the mass of the entire column above it because the effect is reduced by the column being replaced by the fuid surrounding it an so not producing any down force; and the further away from the wing the effect is less by square of the distance. In diagram 2 the square A, if removed will be replaced by the volume of (B) from above represented by the exponential curve $y=x^2$ and the volume being the integral, area $=\int x^2$.

4.Cavitation.

The fluid moving along FA will continue to move in a straight line unless acted on by another force (Newton's first law) and in this case the other force is gravity so the fluid moves along the parabola AIC leaving a spa below. This is called cavitation. [4].

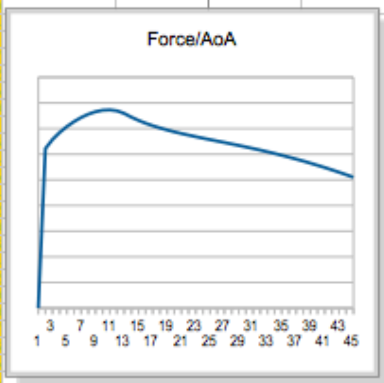
The space created represents an area which does not impart any moment of force to the foil and this loss needs to be subtracted from the total force.

5. Calculating the results:

To investigate the results of this theorectial system and test its validity, a spreadsheet was created so each step in the process could be presented and understood and altered as necessary. And furthermore, then subject it to variables of foil size, fluid velocity and AoA and see how the system responds. Diagram (3). Bear in mind that this system is theorectical and is not yet designed to produce any real values only relative values; so any constants are omitted ie fluid density, span of the foil etc. I was looking to see if, at this stage, it accurately modelled real world results from other sources [1] [2]

	A	B	C	D	E	F	G	H	I	J
1	AOA	Velocity	Foil chord	tan	Simultaneous = x (d2*10)	simultaneous y = b2^2*g	range	Range limited to chord	range limit of x	Area of Δ above foil
2	0	5	0	0.000	0.000	0.000	0.000	0.0000	0.0000	0.000000
3	2	5	1	0.035	0.070	0.002	0.070	0.0699	0.0698	0.017439
4	4	5	1	0.070	0.140	0.010	0.140	0.1402	0.1399	0.034793
5	6	5	1	0.105	0.210	0.022	0.211	0.2114	0.2102	0.051978
6	8	5	1	0.141	0.281	0.040	0.284	0.2838	0.2811	0.068909
7	10	5	1	0.176	0.353	0.062	0.358	0.3581	0.3527	0.085505
8	12	5	1	0.213	0.425	0.090	0.435	0.4346	0.4251	0.101684
9	14	5	1	0.249	0.499	0.124	0.514	0.5139	0.4987	0.117368
10	16	5	1	0.287	0.573	0.164	0.597	0.5966	0.5735	0.132480
11	18	5	1	0.325	0.650	0.211	0.683	0.6833	0.6498	0.146946
12	20	5	1	0.364	0.728	0.265	0.775	0.7747	0.7279	0.160697
13	22	5	1	0.404	0.808	0.326	0.872	0.8715	0.8081	0.173665
14	24	5	1	0.445	0.890	0.396	0.975	0.9747	0.8905	0.185786
15	26	5	1	0.488	0.975	0.476	1.085	1.0000	0.8988	0.197003
16	28	5	1	0.532	1.063	0.565	1.204	1.0000	0.8829	0.207259
17	30	5	1	0.577	1.155	0.667	1.333	1.0000	0.8660	0.216506
18	32	5	1	0.625	1.250	0.781	1.474	1.0000	0.8480	0.224699
19	34	5	1	0.675	1.349	0.910	1.627	1.0000	0.8290	0.231796
20	36	5	1	0.727	1.453	1.056	1.796	1.0000	0.8090	0.237764
21	38	5	1	0.781	1.563	1.221	1.983	1.0000	0.7880	0.242574
22	40	5	1	0.839	1.678	1.408	2.191	1.0000	0.7660	0.246202
23	42	5	1	0.900	1.801	1.621	2.423	1.0000	0.7431	0.248630
24	44	5	1	0.966	1.931	1.865	2.685	1.0000	0.7193	0.249848
25	46	5	1	1.036	2.071	2.145	2.981	1.0000	0.6947	0.249848
26	48	5	1	1.111	2.221	2.467	3.320	1.0000	0.6691	0.248630
45	50	5	1	1.192	2.384	2.841	3.708	1.0000	0.6428	0.246202
46	52	5	1	1.280	2.560	3.277	4.158	1.0000	0.6157	0.242574
47	54	5	1	1.376	2.753	3.789	4.683	1.0000	0.5878	0.237764
48	56	5	1	1.483	2.965	4.396	5.303	1.0000	0.5592	0.231796
49	58	5	1	1.600	3.201	5.122	6.040	1.0000	0.5299	0.224699
50	60	5	1	1.732	3.464	6.000	6.928	1.0000	0.5000	0.216506
51	62	5	1	1.881	3.761	7.074	8.012	1.0000	0.4695	0.207259
52	64	5	1	2.050	4.101	8.407	9.354	1.0000	0.4384	0.197003
53	66	5	1	2.246	4.492	10.089	11.044	1.0000	0.4067	0.185786
54	68	5	1	2.475	4.950	12.252	13.214	1.0000	0.3746	0.173665
55	70	5	1	2.747	5.495	15.097	16.066	1.0000	0.3420	0.160697
56	72	5	1	3.078	6.155	18.944	19.919	1.0000	0.3090	0.146946
57	74	5	1	3.487	6.975	24.324	25.304	1.0000	0.2756	0.132480
58	76	5	1	4.011	8.022	32.173	33.158	1.0000	0.2419	0.117368
59	78	5	1	4.705	9.409	44.267	45.256	1.0000	0.2079	0.101684
60	80	5	1	5.671	11.343	64.327	65.319	1.0000	0.1736	0.085505
61	82	5	1	7.115	14.231	101.257	102.252	1.0000	0.1392	0.068909
62	84	5	1	9.514	19.029	181.046	182.044	1.0000	0.1045	0.051978
63	86	5	1	14.301	28.601	409.018	410.017	1.0000	0.0698	0.034793
64	88	5	1	28.636	57.273	1640.070	1641.070	1.0000	0.0349	0.017439
65										
66										
67						Velocity	5.00			
68						Chord	1			
69										

	K	L	M	N	O	P	Q	R	S	T
1	Area of Δ above range	Area under curve v x^3/3G Limited	Difference J2-K2 Bubble Limited	Fluid Moving	Equivalent chord	Area created above H	Equivalent chord of J	Total length	Force of under wing sin(ø)	Total force
2	0.000000	0.000000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.00
3	0.000085	0.000057	0.0000	0.0174	0.9992	0.0008	0.2096	1.2087	0.0349	1.24
4	0.000684	0.000456	0.0002	0.0346	0.9967	0.0021	0.2481	1.2448	0.0698	1.31
5	0.002322	0.001548	0.0008	0.0512	0.9925	0.0039	0.2726	1.2651	0.1045	1.37
6	0.005552	0.003701	0.0019	0.0671	0.9865	0.0058	0.2898	1.2763	0.1392	1.42
7	0.010964	0.007310	0.0037	0.0819	0.9784	0.0078	0.3021	1.2805	0.1736	1.45
8	0.019207	0.012804	0.0064	0.0953	0.9680	0.0098	0.3105	1.2785	0.2079	1.49
9	0.030999	0.020666	0.0103	0.1070	0.9550	0.0117	0.3154	1.2703	0.2419	1.51
10	0.047154	0.031436	0.0157	0.1168	0.9388	0.0133	0.3168	1.2556	0.2756	1.53
11	0.068605	0.045737	0.0229	0.1241	0.9189	0.0146	0.3149	1.2338	0.3090	1.54
12	0.096433	0.064289	0.0321	0.1286	0.8944	0.0154	0.3092	1.2036	0.3420	1.55
13	0.131904	0.087936	0.0440	0.1297	0.8642	0.0156	0.2994	1.1636	0.3746	1.54
14	0.176514	0.117676	0.0588	0.1269	0.8266	0.0151	0.2849	1.1115	0.4067	1.52
15	0.197003	0.121012	0.0760	0.1210	0.7838	0.0140	0.2669	1.0506	0.4384	1.49
16	0.207259	0.114724	0.0925	0.1147	0.7440	0.0130	0.2500	0.9940	0.4695	1.46
17	0.216506	0.108253	0.1083	0.1083	0.7071	0.0119	0.2342	0.9413	0.5000	1.44
18	0.224699	0.101651	0.1230	0.1017	0.6726	0.0108	0.2193	0.8919	0.5299	1.42
19	0.231796	0.094967	0.1368	0.0950	0.6401	0.0098	0.2051	0.8452	0.5592	1.40
20	0.237764	0.088251	0.1495	0.0883	0.6092	0.0087	0.1917	0.8010	0.5878	1.39
21	0.242574	0.081554	0.1610	0.0816	0.5798	0.0078	0.1789	0.7587	0.6157	1.37
22	0.246202	0.074922	0.1713	0.0749	0.5516	0.0068	0.1666	0.7183	0.6428	1.36
23	0.248630	0.068402	0.1802	0.0684	0.5245	0.0060	0.1549	0.6794	0.6691	1.35
24	0.249848	0.062037	0.1878	0.0620	0.4983	0.0052	0.1436	0.6419	0.6947	1.34
25	0.249848	0.055868	0.1940	0.0559	0.4729	0.0044	0.1327	0.6056	0.7193	1.32
26	0.248630	0.049932	0.1987	0.0499	0.4481	0.0037	0.1223	0.5704	0.7431	1.31
45	0.246202	0.044264	0.2019	0.0443	0.4240	0.0031	0.1123	0.5363	0.7660	1.30
46	0.242574	0.038893	0.2037	0.0389	0.4004	0.0026	0.1027	0.5031	0.7880	1.29
47	0.237764	0.033846	0.2039	0.0338	0.3773	0.0021	0.0934	0.4707	0.8090	1.28
48	0.231796	0.029143	0.2027	0.0291	0.3546	0.0017	0.0846	0.4392	0.8290	1.27
49	0.224699	0.024801	0.1999	0.0248	0.3322	0.0013	0.0761	0.4083	0.8480	1.26
50	0.216506	0.020833	0.1957	0.0208	0.3102	0.0010	0.0680	0.3782	0.8660	1.24
51	0.207259	0.017246	0.1900	0.0172	0.2885	0.0008	0.0604	0.3488	0.8829	1.23
52	0.197003	0.014040	0.1830	0.0140	0.2670	0.0006	0.0531	0.3200	0.8988	1.22
53	0.185786	0.011215	0.1746	0.0112	0.2457	0.0004	0.0462	0.2919	0.9135	1.21
54	0.173665	0.008761	0.1649	0.0088	0.2246	0.0003	0.0397	0.2643	0.9272	1.19
55	0.160697	0.006668	0.1540	0.0067	0.2037	0.0002	0.0336	0.2373	0.9397	1.18
56	0.146946	0.004918	0.1420	0.0049	0.1829	0.0001	0.0280	0.2109	0.9511	1.16
57	0.132480	0.003490	0.1290	0.0035	0.1623	0.0001	0.0228	0.1851	0.9613	1.15
58	0.117368	0.002360	0.1150	0.0024	0.1418	0.0000	0.0180	0.1598	0.9703	1.13
59	0.101684	0.001498	0.1002	0.0015	0.1214	0.0000	0.0138	0.1352	0.9781	1.11
60	0.085505	0.000873	0.0846	0.0009	0.1010	0.0000	0.0100	0.1111	0.9848	1.10
61	0.068909	0.000449	0.0685	0.0004	0.0807	0.0000	0.0068	0.0875	0.9903	1.08
62	0.051978	0.000190	0.0518	0.0002	0.0605	0.0000	0.0041	0.0646	0.9945	1.06
63	0.034793	0.000057	0.0347	0.0001	0.0403	0.0000	0.0020	0.0423	0.9976	1.04
64	0.017439	0.000007	0.0174	0.0000	0.0202	0.0000	0.0006	0.0208	0.9994	1.02



Explanation of the spread sheet by column.

A: Angle of attack.AoA (ø) This is a variable that can be changed.

B: B Velocity in m/s. This is a variable that can be changed via box:

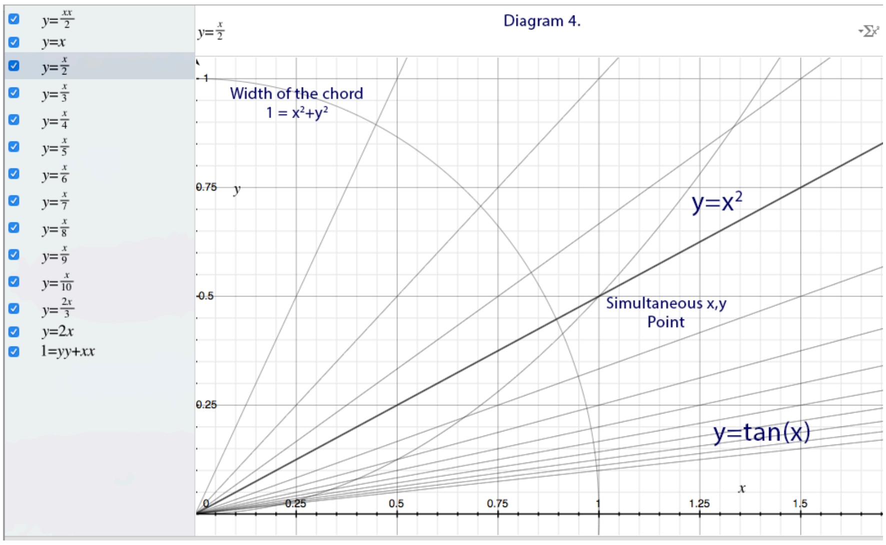
G67

C Chord or wing width, leading edge to trailing edge. This is a variable that can be changed via box:

G68

Changing the velocity and the chord amounts to the same thing as this is only a theorectical overview and not to calculate and actual forces only relative ones.

D. This the tangent of the AoA. It the multiplier which gives the correct slope of the AoA in a graph situation



(see diagram 4) ie y = 0.577x coresponds to an AoA of 30°, tan 30 = 0.577. This then allows the finding of the point at which it crosses the cavitation curve giving the range. (See E onwards)

E. The curve the fluid follows in cavitation (AIC above) is governed its velocity and gravity and this is referred to as the range of cavitation and it is assumed to follow a curve $y = Vx^2/g$. The foil follows a straight line y = tan (x). A simultaneous equation anaylsis will give the x. and y of the intersection of the two lines on a graph. Luckily, I created a simple rule to calculate this without having to resort to simultaneous equation mathematics. The x coordinate is gotten from: x = gtan(ø)/V.
On the spread sheet this becomes:

=D2/(B2/10)

F: This is the y coordinate and is gotten in a similar way to above but this time the simple rule is, $y = g(\tan(ø))^2/V$.

on the spread sheet this becomes:

=(D2^2)/(B2/10)

G: E and F give the x and y coordinates of the intersection so Pythagoras can be used to calculate the hypotenuese in the spread sheet this is expressed as

=SQRT((E2^2)+(F2^2))

H: The range of the cavitation can exceed the width of the chord, any goings-on beyond the chord I deem to be irrelevant and so the range is limited to the chord width. This is achieved simply in the spread sheet as:

=IF(G2<C2;G2;C2)

I: The x coordinate associated with G is needed later, this is gotten by the cosine of the AoA times the range limited by the chord length:

=COS(RADIANS(A2))*H2

J: The area above the chord (AGD) is required later this is given by a formula I made up but probably exists elsewhere but I couldn’t find it, ie: . hypotenuese = √(2.Area/sin(ø)cos(ø))
In the spread sheet this become:

=C2*C2*(SIN(RADIANS(A2))*COS(RADIANS(A2)))/2

K: The area of air above the range (ACJ) is required later, this is gotten by the sin and cos of the hypotenuese mutiplied giving the area of the rectangle and divided by 2 giving the trianglular area ie:

=H2*H2*(SIN(RADIANS(A2))*COS(RADIANS(A2)))/2

L: Next the area under the curve AIC is needed. This in the integral of the formula y = Velocity(range limited)^2/g ie y = Vdist^3/3g. In the spreadsheet this is:

=((I2^3)*B2)/(3*10)

M: So the size of the “bubble” of caviation is AJC - AIC in the spreadsheet this is:

=K2-L2

N: The area of moving fluid is what is needed and this is the triangle above the chord minus the “bubble” that does not extend past the chord. In the spreadsheet this becomes:

=J2-M2

O: The forces created by the moving fuild are calculated in terms of; what width of chord would create that force at that AoA, so as to be comparable with other forces later. So the chord that would create this movement of fluid at this particular AoA needs to be found so this formula is used again… hypotenuese = √(2.Area/sin(ø)cos(ø)).

=SQRT((2*N2)/(SIN(RADIANS(A2))*COS(RADIANS(A2))))

P: The area of fluid moved down from above needs to be found and this will be the intregral of y =x^2, x in this case being the volume which becomes:

=(SQRT(N2)^3/3)

Q: The equivalent chord to shift that fluid needs to be found as in O, this is therefore:

=SQRT(2*P2/(SIN(RADIANS(A2))*COS(RADIANS(A2))))

R: The total effective foil is therefore the sum:

=O2+Q2

S: As stated previously, the simpler forces under the wing are proportional to the sine of the AoA and the chord:

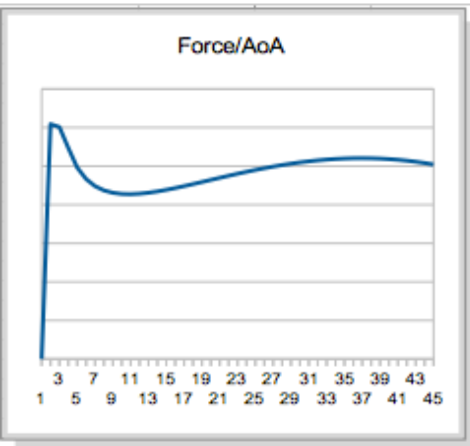
=SIN(RADIANS(A2))*C2

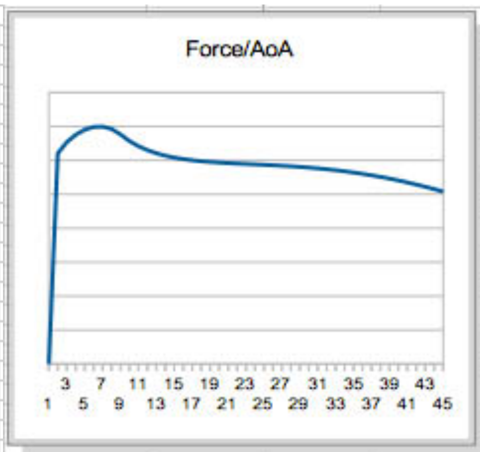
T: The total force on the wing is therefore the sum of the force from the lower and upper surfaces:

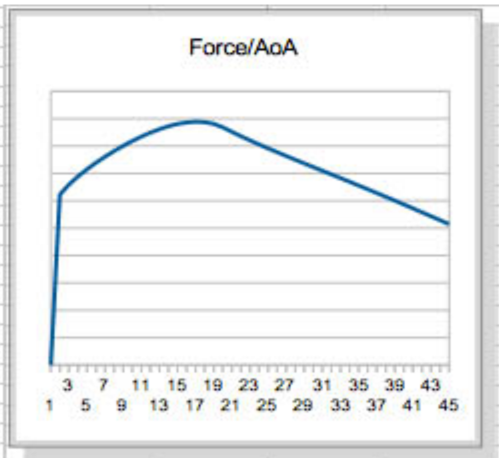
=R2+S2.

G67,68 These are input boxes that vary the velocity and chord universally in columns B and C.

6. Looking at the results.





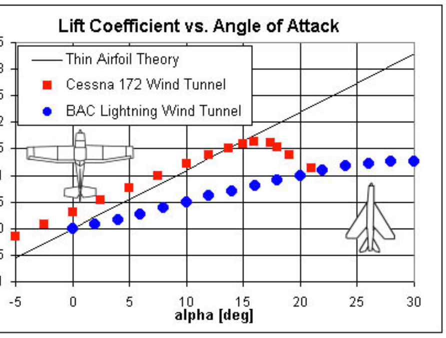


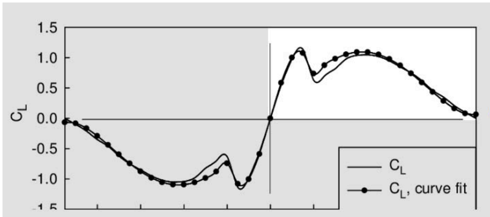
Above are the graphical results of created by the spreadsheet with the input of differing fluid velocities, slow, medium and fast.

Looking at the middle graph first; my mathematics rather lets itself down approaching zero so the first bit of the spreadsheet is induced to read zero at zero angle of attack; from there the information is created by the spreadsheet unhindered; the zero the amount of force rapidly rises to a maximum at about 15º AoA and then declines gradually to 1 at 90º. The left graph shows lowering the speed (or narrowing the chord) causes this be exaggerated as cavitation is less pronounced. Raising the speed the right graph, causes the high point to be pushed towards a higher AoA.

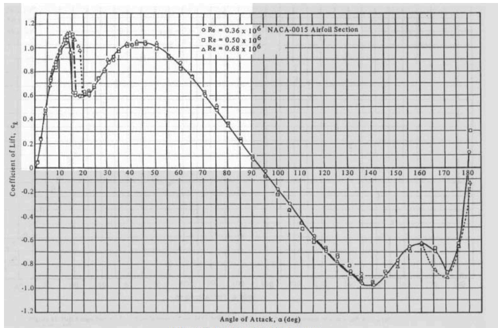
7. Comparison with other data.

There is no data which relates to this exactly because other sources always measure the lift and drag components which are more relevant to their requirements, however, they are relatable.

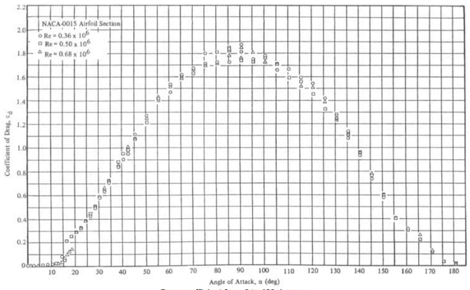




Lift coefficient and curve fit for NACA0015 airfoil for kite's wing. Angle of attack, [deg]-80-60-40-20 0 20 40 60 80



Lift coefficient from 0 to 180 degrees



Drag coefficient from 0 to 180 degrees

Here are some data sets from wind tunnel test from other sources, [1] [2] [3] the areas greyed out are for angles greater than 90º. They are concerned with lift and coefficients of lift so the graphs tend to go from zero to zero, that being the coefficient of lift at zero degrees of AoA and 90º. The latter two graphs show evidence of the same "kink" in the graph and the overall shape of the graph is comparable especially if you add in the drag vector on the bottom graph. And the first shows evidence of the change that occurs with change in velocity or chord width (the comparison between the model Cessna and Lightening wings in a wind tunnel); this concurs with the difference between the spreadsheet graphs, middle and right, above. Additionally, the difference from standard Thin Airfoil Theory can be seen as well.
What may be intersting to sailors is the data also indicates more force maybe created downwind by not having a sail at 90º but at a lesser angle where the downwind component is greater, especially as the wind increases or foil width is large like a spinaker or foresail.

7. Moving on:

I think I have found a way of approaching the problem on how to calculate the predicted force created by a foil at all angles of attack. It is not complete and I can see a few areas that need more detail and would like to adapt it to to give results in Newtons. I would like to see my results compared to some physical results and I intend to create some small apparatus that can do that when I get home. The spreadsheet is available to download and every one is welcome to play with it, point out my errors and enhance it.

References

[1] "Aerodynamic Characteristics of Seven Symmetrical Airfoil Sections Through 180-Degree Angle of Attack For Use In Aerodynamic Analysis of Vertical Axis Wind Turbines" by R.E. Sheldahl and P.C. Klimas

[2] https://www.grc.nasa.gov/WWW/K-12/WindTunnel/Activities/lift_formula.html

[3] https://en.wikipedia.org/wiki/Drag_%28physics%29

[4] Cavitating Flow over a Mini Hydrofoil *

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[5] https://www.researchgate.net/publication/308400130_Modeling_and_Simulation_of_Tethered_Undersea_Kites

[6] http://www.aerospaceweb.org/question/aerodynamics/q0136.shtml

[7] http://www.aerospaceweb.org/question/airfoils/q0150b.shtml

[8] "Aerodynamic Characteristics of Seven Symmetrical Airfoil Sections Through 180-Degree Angle of Attack For Use In Aerodynamic Analysis of Vertical Axis Wind Turbines" by R.E. Sheldahl and P.C. Klimas.

[9] https://support.apple.com/en-gb/guide/grapher/gcalb3dec608/mac

[10] https://www.openoffice.org/